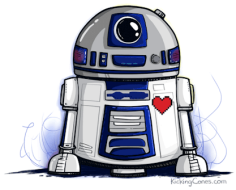
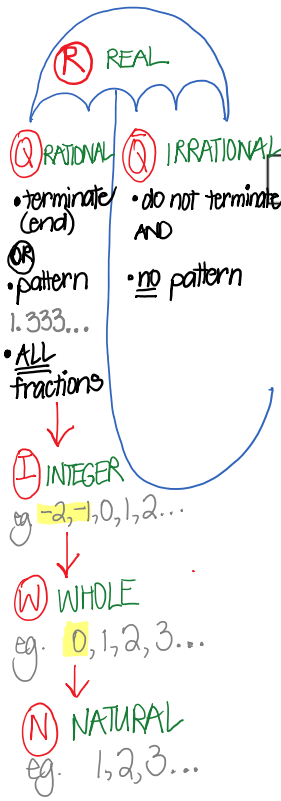




span of x

span of y

Relations Lesson #4:
Domain and Range

Number Systems Review

Number systems were studied in previous math courses. Recall the symbols and letters used by completing the following.

a) Each letter below is represents a number system. Write the name of the number system. $\overline{Q}$ has been completed.

N W ✓ I ✓ Q $\overline{Q}$ Irrational Numbers R All #s

b) The symbol $\in$ represents the phrase "

c) The brackets { } represents the phrase "

d) Recall the following by reading following notes regarding number systems.

Natural Numbers

- The natural numbers (counting numbers) are 1, 2, 3, 4, etc.
- There are an infinite number of natural numbers.
- The set of all natural numbers can be written as $N = \{1, 2, 3, 4, \ldots\}$.
- The number 11 belongs to the set of natural numbers and we can write $11 \in N$.

Whole Numbers

- The whole numbers are the natural numbers together with 0.
- The set of all whole numbers can be written as $W = \{0, 1, 2, 3, \ldots\}$.
- $6 \in N$ but $\frac{1}{2} \notin W$.

Integers

- The integers consist of positive and negative natural numbers, and zero.
- The set of all integers can be written as $I = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.
- In some texts Z or J is used instead of I.

Rational Numbers

- Rational numbers are those numbers which can be expressed as a ratio between two integers, e.g. the fractions $\frac{1}{4}$ and $-\frac{2}{3}$ are both rational numbers. All the integers are included in the rational numbers since any integer can be written as $\frac{x}{1}$ e.g. $5 = \frac{5}{1}$.
- The set of rational numbers can be written as $Q = \left\{\frac{a}{b}, \text{ where } a, b \in I,\ b \neq 0\right\}$.

Irrational Numbers

- The set of irrational numbers can be written as $\overline{Q}$ = {non-terminating and non-repeating decimals} e.g. $\pi \in \overline{Q}$, $\sqrt{2} \in \overline{Q}$.

Real Numbers

- The real numbers consists of all rational numbers and irrational numbers.
- The set of all real numbers can be written as $R = \{Q \text{ and } \overline{Q}\}$.
- In some texts $\Re$ or Re are used instead of R.

Complete Assignment Question #1 and #2

Domain and Range

The **domain** of a relation is the set of all possible values which can be used for the **input** of the **independent variable** (x).

The **range** of a relation is the set of all possible values of the **output** of the **dependent variable** (y).

In lesson 2 on page 235 we described the relation in each of the following forms

- in words
- a table of values
- a set of ordered pairs
- a mapping (or arrow) diagram
- an equation
- a graph

In this lesson we will study the domain and range given in any of these forms.

Class Ex. #1

List the domain and range of the following set of ordered pairs.

a) (1, 2), (0, 5), (3, 8), (5, 9), (−3, 2) **b)** (3, 3), (0, 3), (−3, 3), (2, 9), (−8, 3)

a) domain: 1, 0, 3, 5, −3 → −3, 0, 1, 3, 5 range: 2, 5, 8, 9

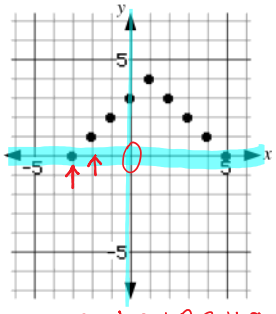
b) domain: −8, −3, 0, 2, 3 range: 3, 9

Note:
① order low → high
② no duplicates
③ ● means ≤ (includes point)
○ means < (does not include point)
④ all smooth connected lines (like Ⓑ & Ⓒ) also need one more thing included
XER (x is all Real #s)
YER (y is all Real #s)

Class Ex. #2

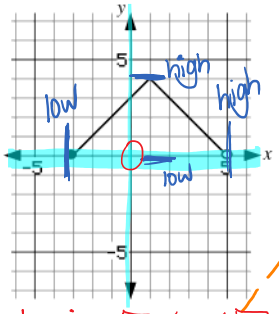
In each case, state the domain and range of the relation represented by the graph.

a)



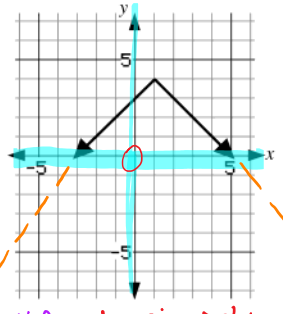
domain: −3, −2, −1, 0, 1, 2, 3, 4, 5
range: 0, 1, 2, 3, 4

b)



domain: −3 ≤ x < 5, XER (−3 low, 5 high)
range: 0 ≤ y ≤ 4, YER (0 low, 4 high)

c)

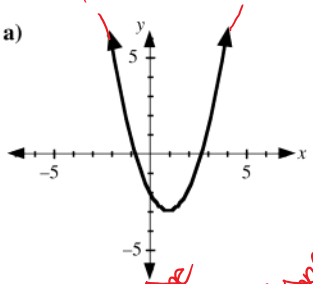


domain: none ≤ x ≤ none, XER
range: none ≤ y ≤ 4, YER

Class Ex. #3

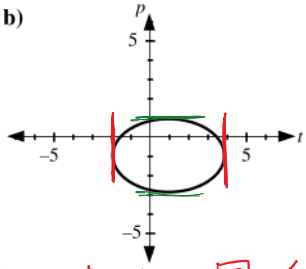
State the domain and range of the following relations.

a)



domain: none ≤ x ≤ none, XER

b)

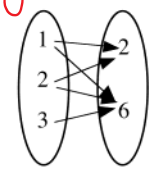


domain: −2 ≤ x ≤ 4, XER

domain: $-5 \leq x \leq$ none, $x \in R$ range: $-3 \leq y \leq$ none, $y \in R$

domain: $-2 \leq x \leq 4$, $x \in R$ range: $-3 \leq y \leq 1$, $y \in R$

c)



d) A circle with centre $(-2, 3)$ and a radius of 3.

a) Draw the graph of a relation which has domain $x \in R$, range $\{y \mid y \leq 2, y \in R\}$ and

i) only one x-intercept **ii)** two x-intercepts

b) Explain why it is not possible to draw a graph which has domain $x \in R$, range $\{y \mid y \leq 2, y \in R\}$ and no x-intercepts.

A high school football team is hosting a banquet to celebrate winning the championship.
The caterer charges a set up fee of \$500 plus \$20 per person.
The equation $C = 500 + 20n$ represents the cost of hosting the banquet for n people.

a) Make a table of values with 8 entries for a minimum of 100 and a maximum of 500 people.

n	C

b) Plot the eight ordered pairs from **a)** on the grid.

c) If all possible ordered pairs from **b)** were plotted on the grid, state the domain and range of the relation and explain why there are restrictions on both.

Complete Assignment Questions #3 - #12

Assignment

rational irrational

#1; 3ACEF; 4ABCEH

/10 mks

1. Use check marks to indicate all the sets to which each number belongs.

	N	W	I	Q	Q'	R
⭐ $\frac{1}{3}$				✓		✓
⭐ 123 983	✓	✓	✓	✓		✓
-2						
7.534						
9.5						

7.534		
9.5		
$\sqrt{75}$		
$-\pi$		
$-355/113$		
$-\sqrt{49}$		
0.000005		
$2.232425...$		

Put check marks in the boxes that each number is part of!

2. The addition of ANY two **natural** numbers is still a natural number.
Does this property hold true for the operations of subtraction, multiplication, or division (except for zero)?

Investigate for whole numbers, integers, rational numbers, and real numbers and write your conclusions.

3. State the domain and range of each relation.

a) $(2,3),\ (0,2),\ (4,8),\ (-1,8),\ (-3,1)$ **b)** $(-3,3),\ (0,-5),\ (-3,3),\ (5,-2),\ (-8,1)$

c)

Input (x)	Output (y)
0	3
2	4
4	5
6	3

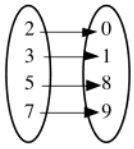
d)

Input (x)	Output (y)
2	3
0	4
–3	5
2	6

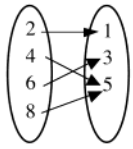
e)

Input (x)	Output (y)
1	5
–1	5
3	5
7	5

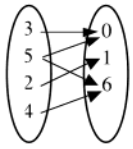
f)



g)

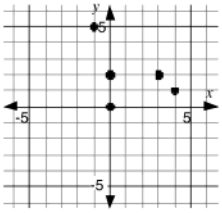


h)

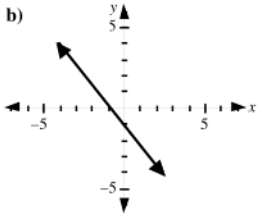


4. State the domain and range for each relation.

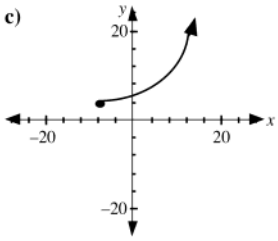
a)



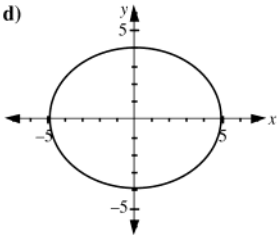
b)



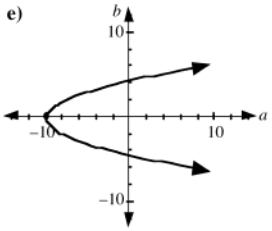
c)



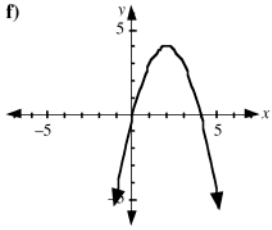
d)



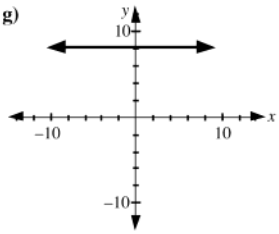
e)



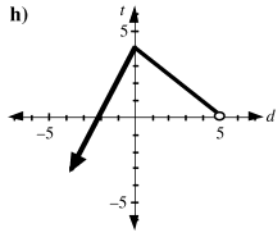
f)



g)



h)



5. In each case a relation is graphed on a grid. State the domain and range of the relation if the graph is

a) a circle whose centre is located at $(-1, 12)$ and with a radius of 5 units.

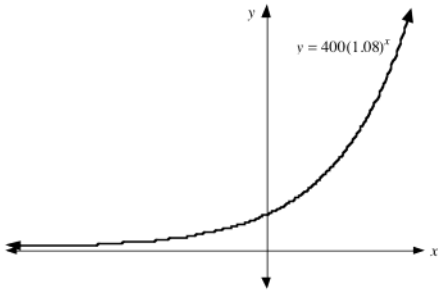
b) a circle with centre $(-3, -5)$ and diameter 40 units.

c) a rectangle with vertices $A(-8, 10)$, $B(-8, -2)$, $C(7, -2)$, and $D(7, 10)$.

d) a triangle with vertices $T(-50, -75)$, $U(-35, -25)$, and $V(-65, -25)$.

6. The graph of the relation $y = 400(1.08)^x$ is shown on the grid.

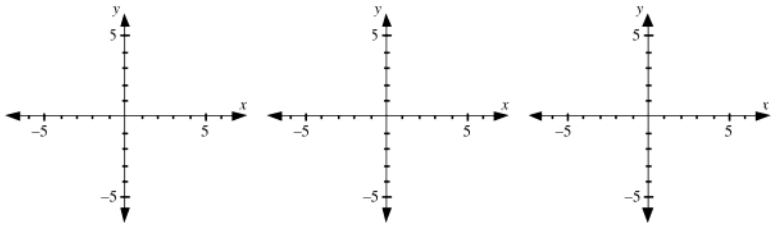
a) State the domain, range, and y-intercept of the relation.



b) The relation $A = 400(1.08)^t$ represents the amount of money when an original investment of $\$400$ is compounded annually at 8% for a period of t years. State the domain and range of this relation, and explain why the answer is different from **a)**.

7. In each case draw a graph on the domain of real numbers which could represent a linear relation with

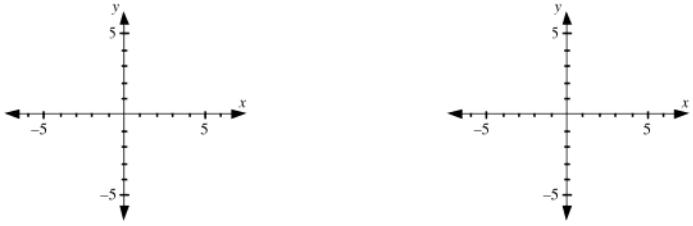
a) one x-intercept **b)** no x-intercept **c)** an infinite number of x-intercepts.



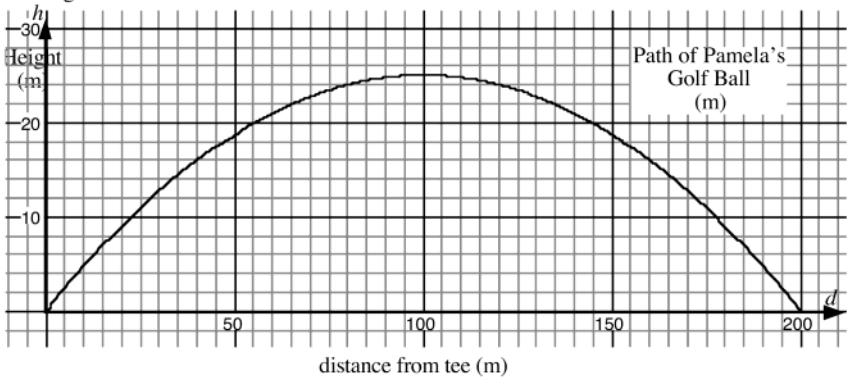
8. In each case draw a graph of a non-linear relation with

a) domain $x \in R$, range $y \geq -3, y \in R$ two x-intercepts and one y-intercept

b) domain $x \in R$, range $y \geq -3, y \in R$ one x-intercept and one y-intercept.



9. The graph shows the flight of Pamela's golf ball from the tee to a sand trap at the edge of the green.



a) State the h-intercept and the d-intercepts of the graph, and explain their significance in relation to the question.

b) State the maximum height of the golf ball, and explain its relevance to the domain or range of the relation.

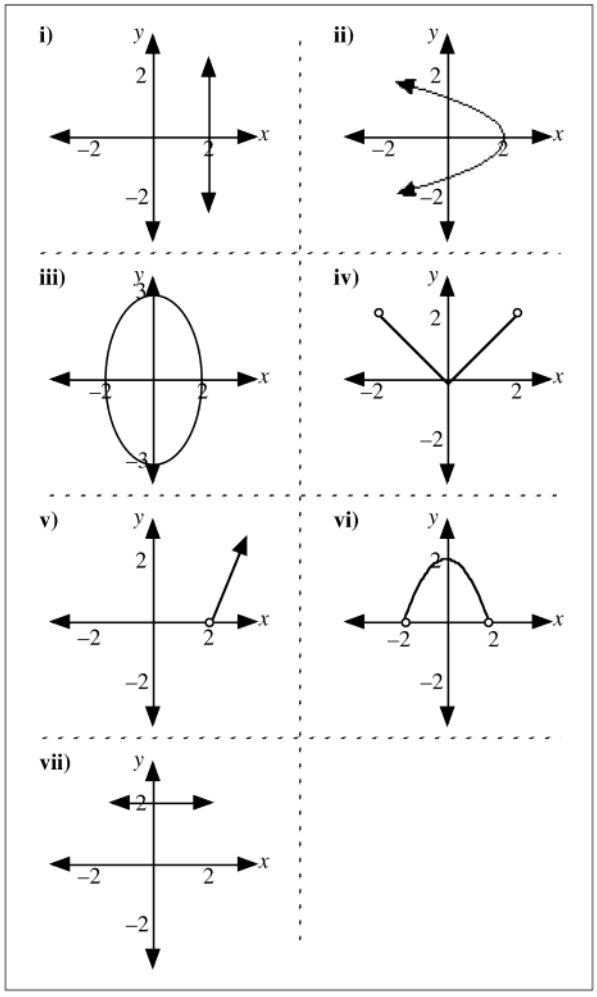
c) State the domain and range of the relation.

d) From the graph, estimate the horizontal distance the ball has traveled when it is 20 m in the air. Explain why there are two answers.

e) Using the graph, estimate the height of the golf ball when the horizontal distance from the tee is 80 m.

f) Give a brief description of the relationship between the height of the golf ball and the horizontal distance from the tee.

10. Match each graph with the domain from A to F. Each domain may be used once, more than once, or not at all.



A. $x = 2$

B. $x < 2$

C. $x \leq 2$

D. $x > 2$

E. $x \geq 2$

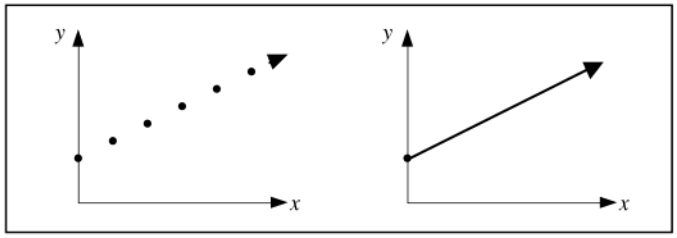
F. $-2 < x < 2$

G. $-2 \leq x \leq 2$

H. $2 < x < -2$

I. $x \in R$

Multiple Choice **11.** The graphs of two relations are shown. Which of the following statements is true?



A. The domains are the same, but the ranges are different.
B. The ranges are the same, but the domains are different.
C. The domains are the same, and the ranges are the same.
D. The domains are different, and the ranges are different.

Numerical Response **12.** The relation between the distance traveled, d km, and the cost, C dollars, of renting a truck is given by the formula $C = 60 + 0.27d$. The domain of the relation can be expressed in the form $d \geq x$, and the range can be expressed in the form $C \geq y$. Write the value of y in the first two boxes and the value of x in the last two boxes.

(Record your answer in the numerical response box from left to right)

Answer Key

1. See chart below

	N	W	I	Q	$\overline{Q}$	R
$\frac{1}{3}$				√		√
123 983	√	√	√	√		√
−2			√	√		√
$7.5\overline{34}$				√		√
9,5				√		√
$\sqrt{75}$					√	√
$-\pi$					√	√
$-\frac{355}{113}$				√		√
$-\sqrt{49}$			√	√		√
0.000005				√		√
2.232425...					√	√

2. See below.

- For natural numbers and whole numbers, the property is true for addition and multiplication only.
- For integers, the property is true for addition, subtraction, and multiplication.
- For rational numbers and real numbers, the property is true for addition, subtraction, multiplication, and division.

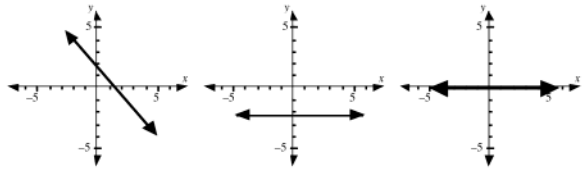
3. **a)** $D = \{2, 0, 4, -1, -3\}$ $R = \{3, 2, 8, 1\}$ **b)** $D = \{-3, 0, 5, -8\}$ $R = \{3, -5, -2, 1\}$ **c)** $D = \{0, 2, 4, 6\}$ $R = \{3, 4, 5\}$ **d)** $D = \{2, 0, -3\}$ $R = \{3, 4, 5, 6\}$
e) $D = \{1, -1, 3, 7\}$ $R = \{5\}$ **f)** $D = \{2, 3, 5, 7\}$ $R = \{0, 1, 8, 9\}$ **g)** $D = \{2, 4, 6, 8\}$ $R = \{1, 3, 5\}$ **h)** $D = \{3, 5, 2, 4\}$ $R = \{0, 1, 6\}$

4. **a)** $D = \{-1, 0, 3, 4\}$ $R = \{5, 0, 2, 1\}$ **b)** $D = \{x \in R\}$ $R = \{y \in R\}$ **c)** $D = \{x \geq -8, x \in R\}$ $R = \{y \geq 4, y \in R\}$ **d)** $D = \{-5 \leq x \leq 5, x \in R\}$ $R = \{-4 \leq y \leq 4, y \in R\}$
e) $D = \{a \geq -10, a \in R\}$ $R = \{b \in R\}$ **f)** $D = \{x \in R\}$ $R = \{y \leq 4, y \in R\}$ **g)** $D = \{x \in R\}$ $R = \{8\}$ **h)** $D = \{d < 5, d \in R\}$ $R = \{t \leq 4, t \in R\}$

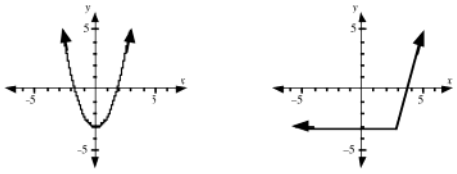
5. **a)** $D = \{-6 \leq x \leq 4, x \in R\}$ $R = \{7 \leq y \leq 17, y \in R\}$ **b)** $D = \{-23 \leq x \leq 17, x \in R\}$ $R = \{-25 \leq y \leq 15, y \in R\}$
c) $D = \{-8 \leq x \leq 7, x \in R\}$ $R = \{-2 \leq y \leq 10, y \in R\}$ **d)** $D = \{-65 \leq x \leq -35, x \in R\}$ $R = \{-75 \leq y \leq -25, y \in R\}$

6. **a)** $D = \{x \in R\}$ $R = \{y > 0, y \in R\}$, y–int is 400
b) $D = \{t \geq 0, t \in R\}$ different from a) because time is never a negative value.
$R = \{A \geq 400, A \in R\}$ different from a) because the amount of money can never be less than \$400.

7. Answers may vary.
a) **b)** **c)**



8. Answers may vary.
a) **b)**



9. **a)** h–int = 0, d–int = 0 and 200. On the tee the ball is on the ground. It returns to ground level 200 m from the tee.
b) max height = 25 m. The maximum height is the upper limit of the range.
c) $D = \{0 \leq d \leq 200, d \in R\}$ $R = \{0 \leq h \leq 25, h \in R\}$
d) 55 m from the tee when the ball is rising and 145m from the tee when the ball is descending.
e) 24 m
f) Starting from a height of 0 m at the tee, the golf ball increases in height to a maximum height of 25m, 100 m from the tee. Then the golf ball starts decreasing in height until it hits the ground 200 m from the tee.

10. **i)** A **ii)** C **iii)** G **iv)** F **v)** D **vi)** F **vii)** I

11. D **12.**

6	0	0	0