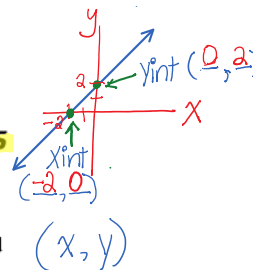


## Relations Lesson #3: x- and y-intercepts and Interpreting Relations

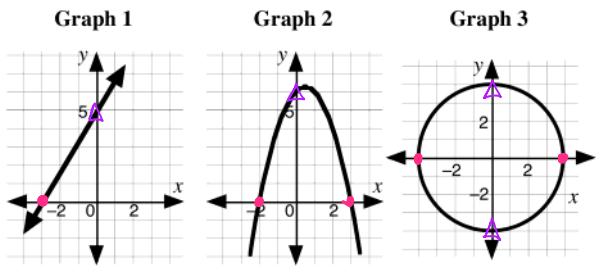


### Review

- a) A **relation** is a connection between two quantities. A relation can be represented graphically by a set of ordered pairs.
- b) The first component of a set of ordered pairs is the x coordinate, also known as the input. Values of the input are values of the independent variable.
- c) The second component of a set of ordered pairs is the y coordinate, also known as the output. Values of the output are values of the dependent variable.

### Exploring x- and y-intercepts

Consider the following graphs.



• = x-int  
Δ = y-int

- a) List the coordinates of the point(s) where each graph crosses the x-axis.
  - Graph 1 crosses the x-axis at  $(-3, 0)$ .
  - Graph 2 crosses the x-axis at  $(-2, 0)$  and  $(3, 0)$ .
  - Graph 3 crosses the x-axis at  $(-4, 0)$  and  $(4, 0)$ .
- b) What do all the points in a) have in common?  
 $y = 0$  ✓
- c) List the coordinates of the point(s) where each graph crosses the y-axis.
  - Graph 1 crosses the y-axis at  $(0, 5)$ .
  - Graph 2 crosses the y-axis at  $(0, 6)$ .
  - Graph 3 crosses the y-axis at  $(0, 4)$  and  $(0, -4)$ .
- d) What do all the points in c) have in common?  
 $x = 0$  ✓

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### x- and y- intercepts of a Graph

Use a graph (last page) or Use Algebra...  
HOW?

The **x-intercept** of a graph is the x-coordinate of the ordered pair where the graph intersects the x-axis. An x-intercept occurs at a point on the graph where the y-coordinate is zero. The x-intercept can be given as a value or as an ordered pair.

The **y-intercept** of a graph is the y-coordinate of the ordered pair where the graph intersects the y-axis. A y-intercept occurs at a point on the graph where the x-coordinate is zero. The y-intercept can be given as a value or as an ordered pair.



1. Given the equation of the graph of a relation:
  - to determine the **x-intercept**, set  $y = 0$  and solve for  $x$ .
  - to determine the **y-intercept**, set  $x = 0$  and solve for  $y$ .
2. The equation of a graph can be written in different forms, all of which are equivalent.

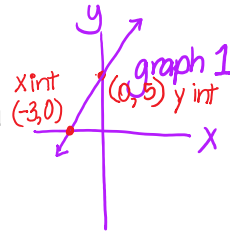
← y



- to determine the **x-intercept**, set  $y=0$  and solve for  $x$ .
- to determine the **y-intercept**, set  $x=0$  and solve for  $y$ .

2. The equation of a graph can be written in different forms, all of which are equivalent.

The equation of Graph 1 on the previous page is  $y = \frac{5}{3}x + 5$ , which can be written as  $3y = 5x + 15$  or  $5x - 3y + 15 = 0$ . Equivalent forms of an equation will be studied in detail in a later unit. For the time being, use the instruction in note 1 to find the  $x$ - and  $y$ -intercepts of the graph of an equation given in any form.



The equation of Graph 1 on the previous page is  $3y = 5x + 15$ . Algebraically determine the values of the  $x$ -intercept and the  $y$ -intercept of Graph 1.

**x-intercept (make  $y=0$ )**

$3y = 5x + 15$

$3(0) = 5x + 15$

$0 = 5x + 15$

$-15 = 5x$

$x = -3$

**Xint  $(-3, 0)$**

**y-intercept (make  $x=0$ )**

$3y = 5x + 15$

$3y = 5(0) + 15$

$3y = 0 + 15$

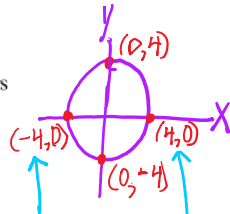
$3y = 15$

$y = 5$

**yint  $(0, 5)$**



The equation of Graph 3 on the previous page is  $x^2 + y^2 = 16$ . Calculate the  $x$ -intercept and the  $y$ -intercept of the graph of  $x^2 + y^2 = 16$ . Give the answers as ordered pairs.



**x-intercept (make  $y=0$ )**

$x^2 + y^2 = 16$

$x^2 + (0)^2 = 16$

$\sqrt{x^2} = \sqrt{16}$

$x = \pm 4$

**Xint  $(-4, 0)$  and  $(4, 0)$**

**y-intercept (make  $x=0$ )**

$x^2 + y^2 = 16$

$(0)^2 + y^2 = 16$

$\sqrt{y^2} = \sqrt{16}$

$y = \pm 4$

**yint  $(0, -4)$  and  $(0, 4)$**

**Complete Assignment Questions #1 - #3**

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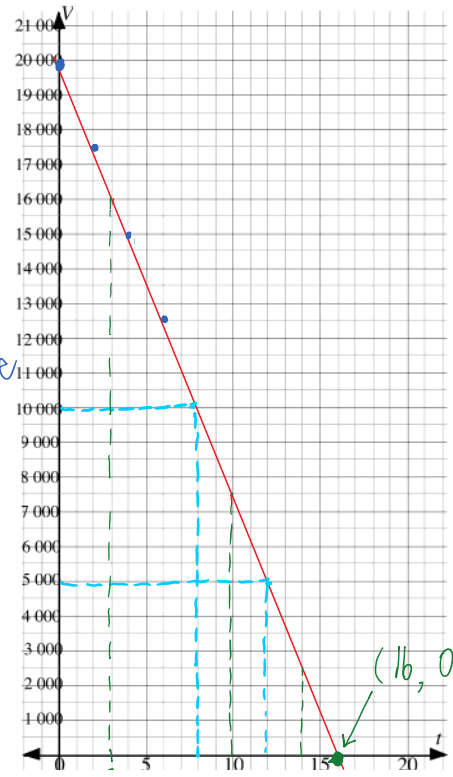
**NOTE!!**  $x^2 = 16$   
 $\hookrightarrow$  2 intercepts!!!



Lisa purchases a new car for \$20 000. The value of the car can be represented by the formula  $V = 20\,000 - 1250t$ , where  $V$  is the value of the car in dollars, and  $t$  is the age of the car in years.

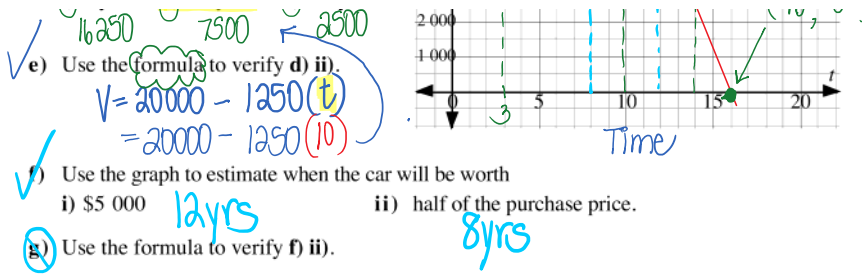
a) Complete the table of values and plot the ordered pairs on the grid.

Input ( $t$ )	Output ( $V$ )	Ordered pair ( $t, V$ )
0	20000	(0, 20000)
2	17500	(2, 17500)
4	15000	(4, 15000)
6	12500	(6, 12500)



Connect the points with a straight line, and extend the line.

- b) What does the ordered pair  $(0, 20\,000)$  represent?  
 new car
- c) Use the graph to estimate the  $t$ -intercept. What does the  $t$ -intercept represent?  
 $t_{int} = 16 \text{ yrs}$
- d) Use the graph to estimate the value of the car after  
 i) 3 years    ii) 10 years    iii) 14 years.  
 16250    7500    2500
- e) Use the formula to verify d) ii).  
 $V = 20000 - 1250(t)$



✓ h) Complete the following statement to describe the relation:  
 The original value of the car is \$ 20 000 . It depreciates in value by \$ 1250 per year and has no value after 16 years.

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Note!!

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In this lesson, using algebra determines the **exact** values for intercepts, etc. whereas using graphs gives an **estimate** for intercepts, etc. In lesson 5 we use the features of a graphing calculator to determine more accurate results from a graph.

In part d)i) we were asked to use the graph to find values lying between given points. This process is called **interpolation**. Extending the graph to predict values outside the plotted points is called **extrapolation**. Examples of extrapolation are d)ii) and d)iii).

Complete Assignment Questions #4 - #9

Assignment # 1 (A) ; 2 (A) ; 3 (A)(C)(E)(H) ; 4 (2 mks)

total : (13 mks)

1. Determine the value of the *y*-intercept of the graph of each equation.

a)  $y = x - 5$

b)  $y = 3x - 15$

c)  $2y + 3x - 12 = 0$

d)  $0.5x - 2.4y + 0.8 = 0$

e)  $2y = x^2 - 60$

f)  $y = 0.001x^2 - 0.001x + 12.44$

2. Determine the value of the *x*-intercept(s) of the graph of each equation.

a)  $y = x - 2$

b)  $y = 2x - 8$

c)  $3y + 2x - 12 = 0$

d)  $0.6x - 2y + 0.5 = 0$

e)  $y = x^2 - 9$

f)  $y = 12 - 3x$

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3. Determine the *x*- and *y*-intercepts of each equation. Answer as ordered pairs.

a)  $y = 4x + 7$

b)  $y = 15 - 6x$

c)  $4x - 2y + 16 = 0$

d)  $y = \frac{x^2}{2} - 18$

e)  $x^2 + y^2 = 25$

f)  $y = 3x$

g)  $y = x^2 + 4$

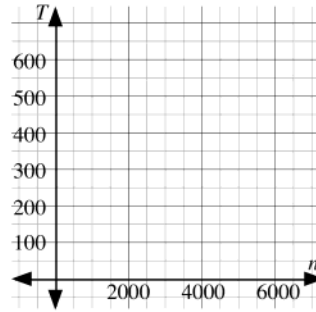
h)  $9x^2 + y^2 = 81$

i)  $9x^2 - y^2 = 81$

4. Triple A Car Rental charges \$100 per rental plus 10¢ per km. The total cost,  $T$ , in dollars of renting the car can be represented by the formula,  $T = 100 + 0.10n$ , where  $n$  is the number of km travelled.

- a) Complete the table of values, and plot the ordered pairs on the grid provided.

Number of km ( $n$ )	Total Rental Cost ( $T$ ) dollars
0	
1000	
3500	
5000	



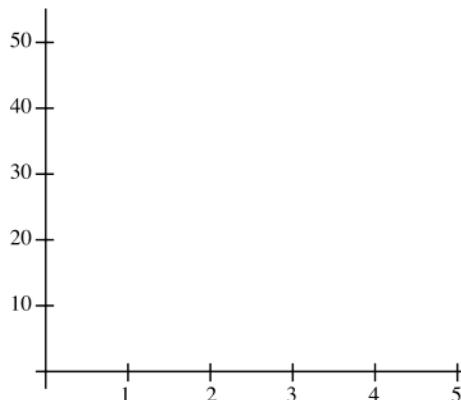
Connect the points with a straight line, and extend the line in both directions.

- b) What does the ordered pair  $(0, 100)$  represent?
- c) Determine the  $n$ -intercept of the graph. Explain why it is not applicable to this problem.
- d) Interpolate from the graph to estimate the cost for a journey of:  
 i) 2000 km                      ii) 4500 km
- e) Use the formula to verify the answers in d).
- f) If the total cost of rental is \$650, use the graph to estimate the number of km travelled.
- g) Verify the answer in f) using the formula.

5. An arrow is shot vertically into the air using a bow. The height, *h* metres, above the ground after *t* seconds, where  $t \geq 0$  is approximated by the equation  $h = -5t^2 + 20t + 25$ .

- a) The maximum height of the arrow is reached after 2 seconds. Calculate the maximum height.
- b) Complete the table of values, and plot the points on the grid. Join the points with a smooth curve, and label the graph.

time (seconds)	height (metres)
0	
1	
2	
3	
4	
5	

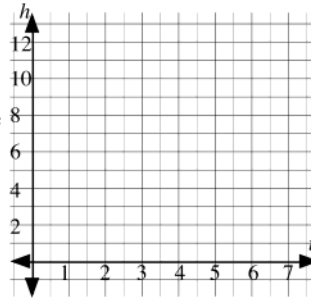


- c) Is this a linear or nonlinear relation?
- d) For how many seconds is the arrow in the air?
- e) What does the *h*-intercept represent in the context of the question?
- f) What does the *t*-intercept represent in the context of the question?
- g) i) Use the graph to estimate the height of the arrow after 1.5 seconds.  
 ii) Use the equation to calculate the exact height of the arrow after 1.5 seconds.
- h) Does it make sense to extend the graph of the relation  $h = -5t^2 + 20t + 25$  further in a downward direction to the left or right? Explain.

6. A candle manufacturer determined that its “Long-Last” candles melted according to the formula  $h = -2t + 12$ , where  $h$  is the height of the candle, in cm, after  $t$  hours.

a) Make a table of values and use this to construct the graph of  $h = -2t + 12$ .

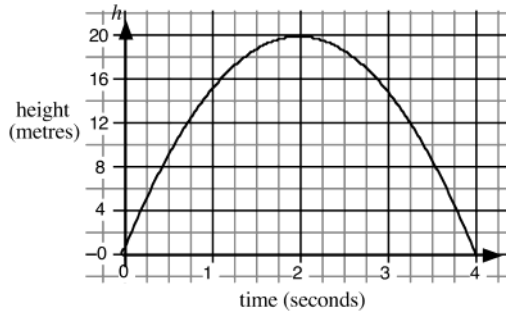
$t$				
$h$				



Use your graph to answer **b** – **e**.

- b) How tall is the candle before it begins to melt?
- c) How many hours will the candle last before it will completely burn out?
- d) How tall will the candle be after burning for 5 hours?
- e) How long will it take for the candle to burn down to a height of 7 cm?
- f) Verify the answers from **b**) - **e**) using the formula.

7. A football is kicked by a student. The graph of the relation between the height of the football above the ground and time is shown. The formula that represents the relation is given by  $h = -4.9t^2 + 19.4t + 0.6$ , where  $h$  is the height in metres above the ground and  $t$  is the time in seconds the football is in the air.



Use the graph to answer a – c:

- Estimate, to the nearest metre, the maximum height of the football above the ground.
  - Estimate how long it takes for the football to reach the ground.
  - Estimate the height, to the nearest metre, of the football when it is in the air for 3 seconds.
  - Use the formula to calculate the exact answer to c).
- e) Calculate the  $h$ -intercept, and describe what it represents in the context of the question.

**Multiple Choice** 8. In which of the following relations does the graph of the relation have  $x$ - and  $y$ -intercepts with equal values?

- $y = x + 8$
- $2x + 2y = 7$
- $2x - 3y + 4 = 0$
- none of the above

**Numerical Response** 9. The graph of the relation  $4x^2 + 9y^2 - 36 = 0$  has  $x$ -intercepts  $a$  and  $b$ , and  $y$ -intercepts  $c$  and  $d$ . The value of the product  $abcd$  is \_\_\_\_\_.

(Record your answer in the numerical response box from left to right)

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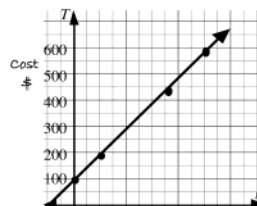
**Answer Key** 1. a) -5 b) -15 c) 6 d)  $\frac{1}{3}$  e) -30 f) 12.44

2. a) 2 b) 4 c) 6 d)  $-\frac{5}{6}$  e)  $\pm 3$  f) 4

3. a)  $x$ -int =  $(-\frac{7}{4}, 0)$ ,  $y$ -int = (0, 7) b)  $x$ -int =  $(\frac{5}{2}, 0)$ ,  $y$ -int = (0, 15) c)  $x$ -int = (-4, 0)  $y$ -int = (0, 8)
- d)  $x$ -int = (6, 0) and (-6, 0),  $y$ -int = (0, -18) e)  $x$ -int = (5, 0) and (-5, 0)  $y$ -int = (0, 5) and (0, -5)
- f)  $x$ -int = (0, 0),  $y$ -int = (0, 0) g) no  $x$ -int,  $y$ -int = (0, 4)
- h)  $x$ -int = (3, 0) and (-3, 0)  $y$ -int = (0, 9) and (0, -9) i)  $x$ -int = (3, 0) and (-3, 0) no  $y$ -int

4. a) see table and graph  
 b) Triple A Car Rental charges a fixed rate of \$100 before any distance is travelled  
 c)  $n$ -int = -1000, distance in this scenario cannot be represented by a negative value  
 d) i) \$300 ii) \$550  
 f) 5500 km

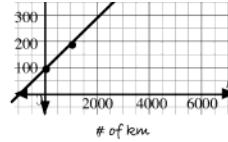
Number of km ( $n$ )	Total Rental Cost ( $T$ ) dollars
0	100
1000	200
3500	450
5000	600





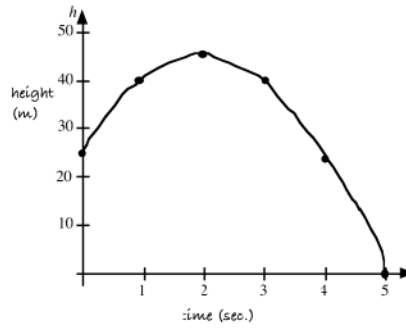
- represented by a negative value  
 d) i) \$300 ii) \$550  
 f) 5500 km

3500	450
5000	600



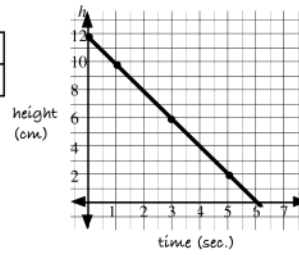
5. a) 45 m  
 b) see table and graph  
 c) non-linear  
 d) 5  
 e) The arrow was fired from a height of 25 m above the ground  
 f) The number of seconds it takes to strike the ground.  
 g) i) approximately 44 m  
 ii) 43.75  
 h) No to the left because time cannot be negative.  
 No to the right because the ground stops the arrow from going further.

time (seconds)	height (metres)
0	25
1	40
2	45
3	40
4	25
5	0



6. a) see table and graph answers may vary  
 b) 12 cm  
 c) 6 hours  
 d) 2 cm  
 e) 2.5 hours

$t$	0	1	3	5
$h$	12	10	6	2



7. a) approx 20 m  
 b) approx 4 seconds  
 c) approx 15 m  
 d) 14.7 m  
 e)  $h - \text{int}$  is 0.6 m.  
 The football was punted 0.6 m above the ground.

8. B      9. 

3	6		
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