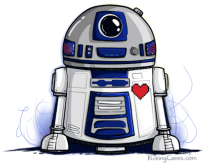


Relations Lesson #2:
Relationships Between Two Quantities

Relations

Much of mathematics involves the search for patterns and relationships between sets of data. Many real life applications of mathematics investigate the relationship between two quantities.

For example:

- the value of a computer is related to its age
- the price of a watermelon is related to its weight
- the time taken for a person to walk to school is related to the walking distance.

In mathematics, a comparison between two sets of elements is called a **relation**.

List one more example of a relation.

Representing the Relationship Between Two Quantities

In the next two units we will consider seven ways in which the relationship between two quantities can be represented.

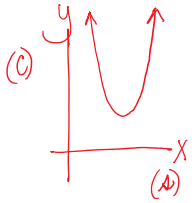
- in words
- a table of values
- a set of ordered pairs
- a mapping (or arrow) diagram
- an equation
- a graph
- function notation (some relations can be represented in this way as in the next unit)

We will use the relation below as an example.

Investigating a Relation

Consider the following relation:

> **"The cost, C (cents per km), of driving a car is related to the speed, s (km/h), at which it is driven."**



We will use this relation to introduce some ideas which will be developed throughout the course of this unit. Our task is to represent this relation in some form.

The example illustrates a relationship between two variables, C and s.
In the statement of the relation, the cost depends on the speed.

C is called the **dependent variable** and s is called the **independent variable**.

When representing a relation, we often regard the values of the independent variable as the **input** and the values of the dependent variable as the **output**.

Before considering how to represent this relation we need some data: we need input values and output values.

The input values make up the **domain** of the relation, and the output values make up the **range** of the relation. These concepts will be discussed in more detail later.

Summary:

X	Y
"s"	"C"
independent variable	dependent variable
inputs	outputs
domain (span of xs)	range (span of ys)

Obviously we would not attempt to collect data for every possible input value (i.e. for every possible speed at which the car could be driven). Suppose that we choose as input values speeds of 20, 30, 40, ...120 km/h. and that the output values are as given in the diagrams below.

1) **Table of Values**

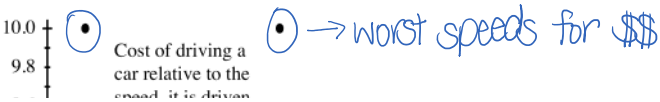
Input (s)	Output (C)
20	10

2) **Ordered Pairs**

(s, C)

$(20, 10)$
$(30, 9.1)$

3) **Graph**



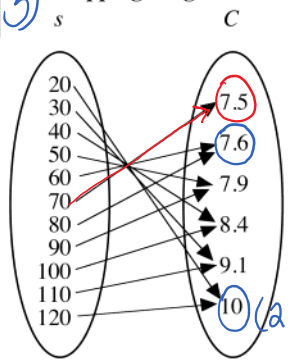
⊙ → worst speeds for $$

Input (s)	Output (C)
20	10
30	9.1
40	8.4
50	7.9
60	7.6
70	7.5
80	7.6
90	7.9
100	8.4
110	9.1
120	10

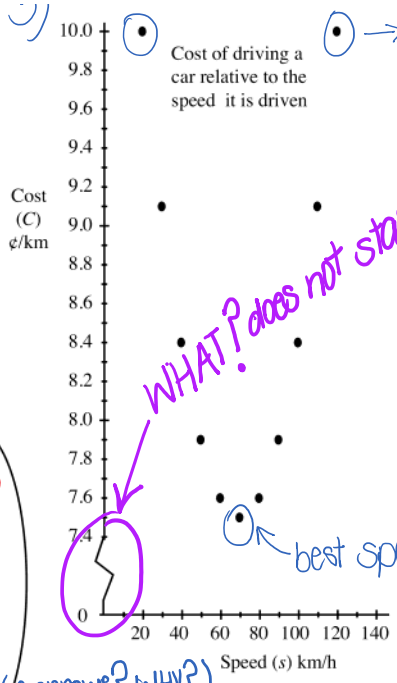
(s, C)

$(20, 10)$
$(30, 9.1)$
$(40, 8.4)$
$(50, 7.9)$
$(60, 7.6)$
$(70, 7.5)$
$(80, 7.6)$
$(90, 7.9)$
$(100, 8.4)$
$(110, 9.1)$
$(120, 10)$

3) **Mapping Diagram**



(2 arrows? WHY?)



→ worst speeds for $$

WHAT? does not start @ 1!

best speed for fuel economy

The diagrams above also illustrate how the information collected can be represented

- as ordered pairs in a table of values
- as a mapping diagram
- graphically

We have only chosen some of the possible input values, however it is obvious that an output value could have been determined for any input value greater than zero and up to the maximum speed of the car. It makes sense then to connect the points on the graph in some way.

Later we will learn how this can be done and how an equation can be determined that best represents the data.

The graph and equation for the relation are given below. Note that the equation is only valid for certain input values which make up the domain of the relation. For example, the equation would not be valid for $s = 5000$!

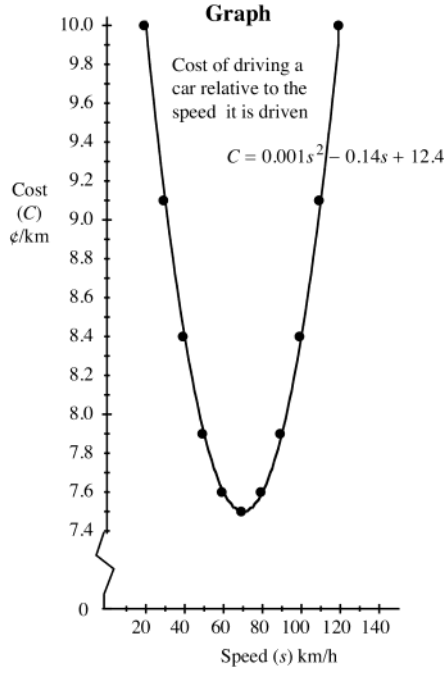
Eq^N used

Equation

$$C = 0.001s^2 - 0.14s + 12.4$$

plotted data

Graph



Independent and Dependent Variables in a Relation

The values of the independent variable represent the inputs, and the corresponding values of the dependent variable are the outputs.

- In an ordered pair, the values of the first coordinate are those of the independent variable and the values of the second coordinate are values of the dependent variable.
- In a table of values, the independent variable is usually given first - either to the left or above the values of the dependent variable.
- In a mapping diagram, the arrows point from the independent variable to the dependent variable.

(x, y)

x	y

$x \to y$

- In a mapping diagram, the arrows point from the independent variable to the dependent variable.
- On a graph, the independent variable is on the horizontal axis, often the x-axis, and the dependent variable is on the vertical axis, often the y-axis.
- In an equation, we usually try to isolate the dependent variable to the left side.

The illustration below uses the equation $y = 3x - 5$ as an example to illustrate the independent and dependent variables of an equation.

$$y = 3x - 5$$

The dependent variable.	The independent variable.
• Values of the dependent variable represent the outputs of the relation.	• Values of the independent variable represent the inputs of the relation.
• Values of the dependent variable are represented by the **second** coordinate of an ordered pair and are on the vertical axis.	• Values of the independent variable are represented by the **first** coordinate of an ordered pair and are on the horizontal axis.

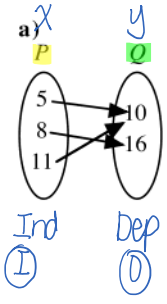
Class Ex. #2

The diagrams show relations expressed in different ways. In each case

i) state the independent variable **ii)** state the dependent variable

iii) list the inputs **iv)** list the outputs

a)



b)

V	A
4	15
10	12
25	15

c) (B, c): (3, 7), (4, 11), (5, 15), (6, 19)

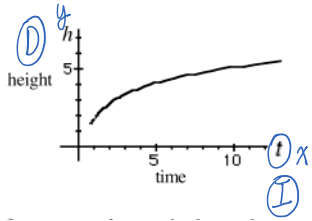
Class Ex. #3

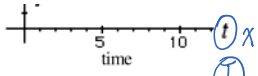
Relations are expressed in different ways below. In each case

i) state the independent variable **ii)** state the dependent variable

a) $C = 2\pi r$

b)





c) The amount of sap, s, obtained from a maple tree is dependent on the time, t, a container is left attached to the maple tree.

Complete Assignment Questions #1 - #4

Investigating Relationships by Plotting Ordered Pairs

In this section we will consider relations defined by an equation, and sketch a graph by plotting ordered pairs using the following steps:

- Make a table of inputs by choosing replacements for the independent variable.
- For each of the input values, calculate the corresponding value (the output) of the dependent variable.
- Plot the ordered pairs on a Cartesian plane.

Consider the relation described by the equation $y = 2x - 5$.

a) Complete the first five rows of the following table of values which shows some of the possible input values.

Input (x)	Output (y)	Ordered pair (x, y)
-2	-9	(-2, -9)
-1	-7	(-1, -7)
0	-5	(0, -5)
1	-3	(1, -3)
2	-1	(2, -1)
6	7	
4	3	

$y = 2x - 5$

$y = 2(-2) - 5 = -9$

$y = 2(-1) - 5 = -7$

$y = 2(0) - 5 = -5$

$y = 2(1) - 5 = -3$

$y = 2(2) - 5 = -1$

b) Plot the ordered pairs in **a)** on the grid provided.

c) Connect the points on the grid, and extend the line in both directions with arrows at both ends.

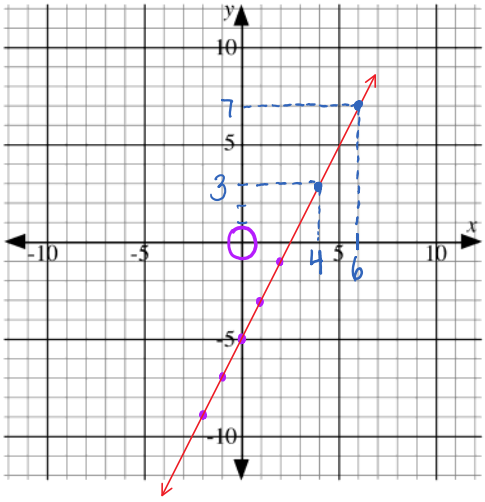
d) Use the graph to determine the value of y when $x = 6$.

e) Use the equation to determine the value of y when $x = 6$, and verify the answer in **d)**.

$y = 2x - 5$

$y = 2(6) - 5$

$= 7$ same answer



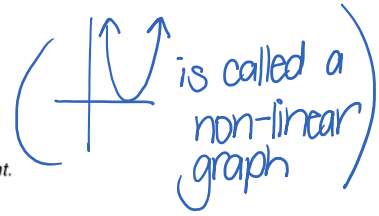
f) Write the value of y when $x = 6$ in the table of values using the first blank space in **a)**.

g) Use the graph to determine the value of x when $y = 3$. Put this information in the last row in **a)**.

g) Use the graph to determine the value of x when $y = 3$. Put this information in the last row in **a)**.

h) Complete the following statement:

This relation is called a LINEAR relation because the graph of the relation is a straight line.

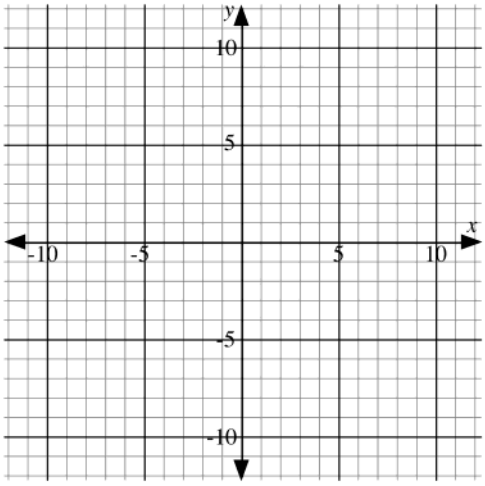


Consider the relation described by the equation $y = x^2 - 6$.

a) Complete the table of values to the right which shows some of the possible input values.

Input (x)	Output (y)	Ordered pair (x, y)
4		
3		
2		
1		
0		
–1		
–2		
–3		

b) Plot the ordered pairs in **a)** on the grid provided.



c) Use the symmetry of the graph or table to predict the value of y when $x = -4$.

d) Use the equation to determine the value of y when $x = -4$, and verify the answer in **c)**.

e) Write the value of y when $x = -4$ in the table of values using the first blank space in **a)**.

f) Connect the points on the grid with a smooth curve.

g) Why do you think this type of relation is called a **nonlinear relation**?

Complete Assignment Questions #5 - #9

Assignment

1-3; 5; 6 /32

1. Complete the following.

a) The mathematical relationship between two quantities is called a _____ .

b) The variable used for inputs in a relation is known as the _____ variable.

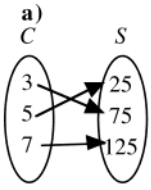
c) The variable used for outputs in a relation is known as the _____ variable.

d) In the equation $A = \pi r^2$, the independent variable is _____, and the dependent variable is ____.

2. The diagrams show relations expressed in different ways. In each case:

i) state the independent and dependent variables **ii)** list the inputs and outputs.

a)



b)

C	n
8	22
20	19
50	35

c)

(f, e): $(2, 3)$, $(-2, 19)$, $(8, 17)$, $(0, 2)$

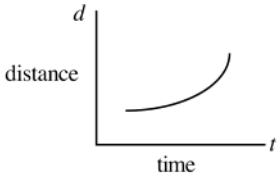
3. For each of the following relations, state

i) the independent variable **ii)** the dependent variable

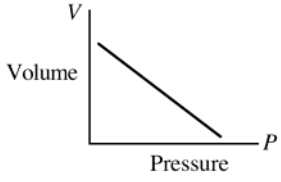
a) $V = \frac{4}{3}\pi r^3$

b) $C = \frac{5}{9}(F - 32)$

c)



d)



e) A truck's value, v, depends on its age, a.

f) The cost, C, of producing business cards is dependent on the number of cards, n, produced.

4. List the different ways a relation may be represented.

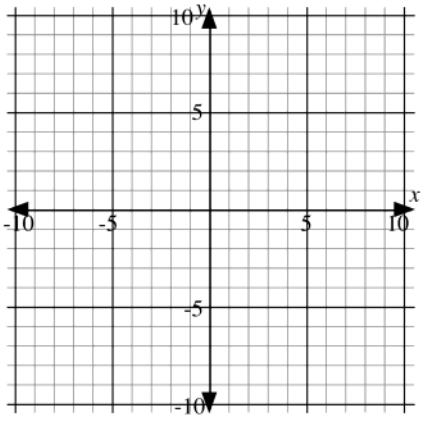
5. Consider the relation described by the equation $y = -x - 2$.

a) Identify the independent and dependent variables.

b) Complete the following table of values.

Input (x)	Output (y)	Ordered pair (x, y)
–3		
–1		
0		
1		

c) Plot the ordered pairs in **b)** on the grid provided.



d) Connect the points on the grid, and then extend the line in both directions with arrows at both ends.

e) Use the graph to determine the value of y when $x = 5$.

f) Use the equation to determine the value of y when $x = 5$, and verify the answer in **e)**.

g) Write the value of y when $x = 5$ in the table of values in **b)**.

h) Use the graph to determine the value of x when $y = 0$. Include this ordered pair in the table of values.

i) Use the graph to determine the value of x when $y = 4$. Include this ordered pair in the table of values.

j) Verify the answer in i) using the equation.

k) Is this a linear or a nonlinear relation?

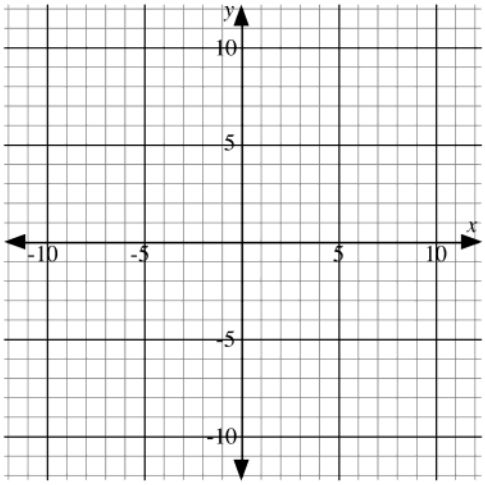
6. Consider the relation described by the equation $y = -0.5x^2 + 8$.

a) Identify the independent and dependent variables.

b) Complete the following table of values.

Input (x)	Output (y)	Ordered pair (x, y)
–6		
–4		
–2		
0		
2		
4		

c) Plot the ordered pairs in **b)** on the grid provided.



d) Use the plotted points and table to predict the value of y when $x = 6$. Plot this point on the grid.

e) Use the equation to determine the value of y when $x = 6$, and verify the answer in **d)**.

f) Connect the points on the grid with a smooth curve.

g) Is this a linear or a nonlinear relation?

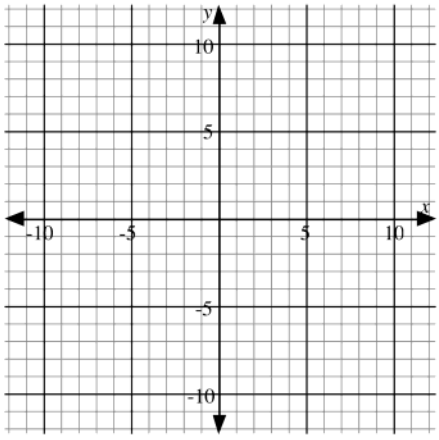
h) Use the graph to determine the values of x when $y = 3.5$. Verify your answer by using the equation.

7. For the following relations:

i) Complete the table of values choosing your own input values where necessary.
ii) Plot the ordered pairs on the grid, and sketch the graph of the relation.
iii) State whether the relation is linear or nonlinear.

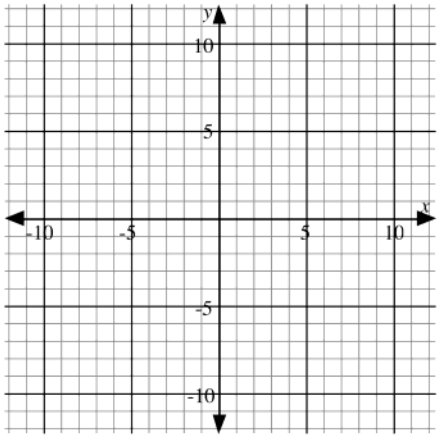
a) $y = -2x + 3$

Input (x)	Output (y)	Ordered pair (x, y)
0		
3		



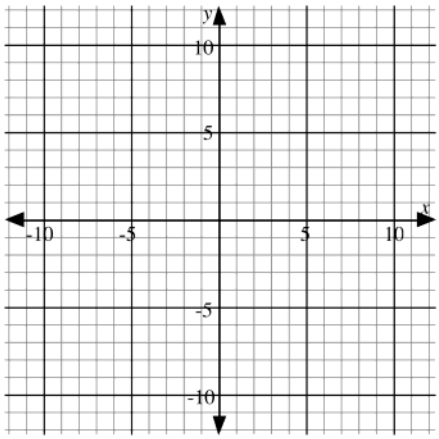
b) $y = 0.5x - 8$

Input (x)	Output (y)	Ordered pair (x, y)
–8		
0		
6		



c) $y = -x^2 + 5$

Input (x)	Output (y)	Ordered pair (x, y)
4		
3		
2		
1		
0		
–1		
–2		
–3		
–4		



Multiple Choice **8.** Which of the following statements is false?

A. The dependent variable is represented on the vertical axis of a Cartesian Plane.
B. The independent variable is represented by the first coordinate of an ordered pair.
C. The outputs of a relation are shown on the horizontal axis of a Cartesian Plane.
D. The independent variable is usually shown on the right side of an equation.

Numerical Response **9.** Consider the relation described by the equation $y = 1.5^{x-2}$. If the input is 4, then the output is _____ .

(Record your answer in the numerical response box from left to right) | | | | |

Answer Key

1. **a)** relation **b)** independent **c)** dependent **d)** r, A

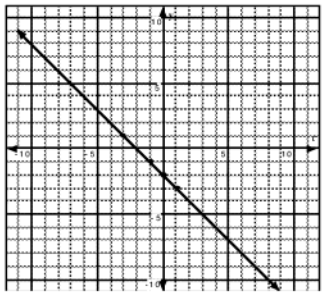
2. **a)** **i)** independent - C; dependent - S **ii)** input - 3, 5, 7; output - 25, 75, 125
b) **i)** independent - C; dependent - n **ii)** input - 8, 20, 50; output - 22, 19, 35
c) **i)** independent - f; dependent - e **ii)** input - 2, –2, 8, 0; output - 3, 19, 17, 2

3. **a)** **i)** independent - r **ii)** dependent - V
b) **i)** independent - F **ii)** dependent - C
c) **i)** independent - time **ii)** dependent - distance
d) **i)** independent - pressure **ii)** dependent - volume
e) **i)** independent - a **ii)** dependent - v
f) **i)** independent - n **ii)** dependent - C

4. words, table of values, set of ordered pairs, mapping, equation, graph, function notation

5. **a)** Independent $\rightarrow x$, Dependent $\rightarrow y$
b) See table below **c), d)** See graph below **e)** –7 **f)** –7

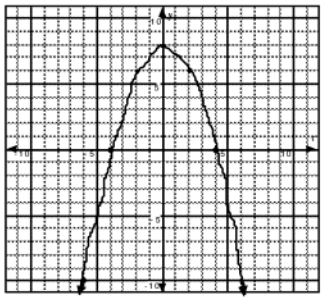
Input (x)	Output (y)	Ordered pair (x, y)
–3	**1**	**(–3, 1)**
–1	**–1**	**(–1, –1)**
0	**–2**	**(0, –2)**
1	**–3**	**(1, –3)**
g) 5	**–7**	**(5, –7)**
h) **–2**	**0**	**(–2, 0)**
i) **–6**	**4**	**(–6, 4)**



k) linear

6. **a)** Independent $\rightarrow x$ Dependent $\rightarrow y$
b) See table below. **c), f)** See graph below. **d)** –10 **e)** $y = -0.5(6)^2 + 8\ y = -10$ **g)** non-linear

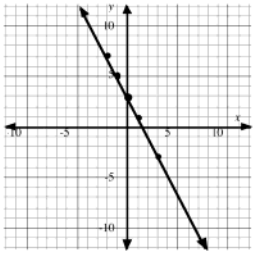
Input (x)	Output (y)	Ordered pair (x, y)
–6	**–10**	**(–6, –10)**
–4	**0**	**(–4, 0)**
–2	**6**	**(–2, 6)**
0	**8**	**(0, 8)**
2	**6**	**(2, 6)**
4	**0**	**(4, 0)**



h) $x = \pm 3$

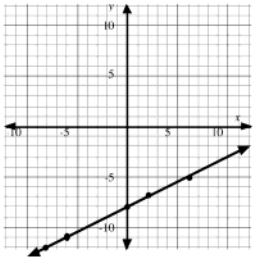
7. a) i) See table below. Inputs may vary. **ii)** See grid below. **iii)** linear

Input (x)	Output (y)	Ordered pair (x, y)
–2	**7**	**(–2, 7)**
–1	**5**	**(–1, 5)**
0	**3**	**(0, 3)**
1	**1**	**(1, 1)**
3	**–3**	**(3, –3)**



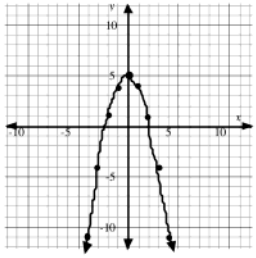
b) i) See table below. Inputs may vary. **ii)** See grid below. **iii)** linear

Input (x)	Output (y)	Ordered pair (x, y)
–8	**–12**	**(–8, –12)**
–6	**–11**	**(–6, –11)**
0	**–8**	**(0, –8)**
2	**–7**	**(2, –7)**
6	**–5**	**(6, –5)**



c) i) See table below. **ii)** See grid below. **iii)** non-linear

Input (x)	Output (y)	Ordered pair (x, y)
4	**–11**	**(4, –11)**
3	**–4**	**(3, –4)**
2	**1**	**(2, 1)**
1	**4**	**(1, 4)**
0	**5**	**(0, 5)**
–1	**4**	**(–1, 4)**
–2	**1**	**(–2, 1)**
–3	**–4**	**(–3, –4)**
–4	**–11**	**(–4, –11)**



8. C **9.**

2	.	2	5